MODEL QUESTION PAPER

06MAT21

Second Semester B.E. Degree Examination

Engineering Mathematics - II

Time: 3 hrs. Max. Marks:100

Note:1. ***Answer any FIVE full questions, choosing at least two from each part***

2. ***Answer all objectives type questions only in first and second writing pages.***

3. ***Answer to the objective type questions should not be repeated.***

Part A

1 a. Select correct answer in each of the following:

i) The curvature at any point on the curve $y=x^3$ is,

A) $\frac{2x}{(1+9x^4)^{3/2}}$ B) $\frac{6x}{(1+9x^4)^{3/2}}$ C) $\frac{5x}{(1+9x^4)^{3/2}}$ D) None of these.

ii) The radius of curvature of a curve in the pedal form is,

A) $r^2\frac{d^2r}{dp^2}$ B) $r\frac{dr}{dp}$ C) $p\frac{dr}{dp}$ D) $r^2\frac{dr}{dp}$

iii) The value of 'C' of the Cauchy's mean value theorem for $f(x)=e^x$ and $g(x)=e^{-x}$ in [4, 5] is,

A) $\frac{5}{2}$ B) $\frac{3}{2}$ C) $\frac{9}{2}$ D) $\frac{1}{2}$

iv) Maclaurin's series expansion of e^x is,

A) $1+x+\frac{x^2}{2!}+\frac{x^3}{3!}+.....$ B) $x-\frac{x^3}{3!}+\frac{x^5}{5!}-.....$ C) $x+\frac{x^3}{3!}+\frac{x^5}{5!}+.....$ D) $1-\frac{x^2}{2!}+\frac{x^4}{4!}...$ (04 Marks)

b. Find the radius of curvature of the curve $x^4+y^4=2$ at the point (1, 1). (04 Marks)

c. State and prove Cauchy's mean value theorem. (06 Marks)

d. Expand $e^{\sin x}$ using Maclaurin's series upto the term containing x^4. (06 Marks)

2 a. Select correct answer in each of the following:

i) $\underset{x\to 0}{\mathrm{Lt}}\frac{a^x-b^x}{x}=$

A) log ab B) $\log\frac{a}{b}$ C) a log b D) Does not exist

ii) The set of necessary conditions for f(x, y) to have a maximum or minimum is,

A) $\frac{\partial f}{\partial x}=0, \frac{\partial f}{\partial y}=0$ B) $\frac{\partial^2 f}{\partial y\partial x}=0$ C) $\frac{\partial^2 f}{\partial x^2}=0$ D) None of these

iii) The rectangular solid of maximum volume that can be inscribed in a sphere is,

A) a cube B) a triangle C) a rectangle D) None of these

iv) In a plane triangle ABC the maximum value of cosAcosBcosC is,

A) $\frac{3}{8}$ B) $\frac{1}{8}$ C) $\frac{5}{8}$ D) $\frac{25}{8}$ (04 Marks)

b. Evaluate $\underset{x\to 0}{\mathrm{Lt}}\, x\log\tan x$. (04 Marks)

c. Expand cosxcosy in powers of x and y upto fourth degree terms. (06 Marks)

d. Find the extreme values of xy subject to the condition $x^2+xy+y^2=a^2$. (06 Marks)

3 a. Select correct answer in each of the following:

i) $\int_1^4\int_3^5 x^2 y\,dy\,dx =$

A) 58 B) 168 C) 100 D) 125

ii) $\int_0^a\int_0^b\int_0^c x^2y^2z^2\,dx\,dy\,dz =$

A) $\frac{a^3b^3c^3}{27}$ B) $\frac{abc}{3}$ C) $\frac{a^2b^2c^2}{27}$ D) $\frac{a^2bc^2}{27}$

iii) The value of $\beta(2, 1)+\beta(1, 2)$ is,

A) 2 B) 1 C) 3 D) 4

iv) The value of $\Gamma\left(\frac{1}{2}\right)$ is,

A) $\sqrt{\pi}$ B) π C) $\frac{\pi}{2}$ D) $\frac{\sqrt{\pi}}{2}$ (04 Marks)

b. Find the area between the parabolas $y^2=4ax$ and $x^2=4ay$. (04 Marks)

c. Evaluate $\int_0^1\int_0^{1-x}\int_0^{1-x-y}\frac{dz\,dy\,dx}{(1+x+y+z)^3}$. (06 Marks)

d. Express $\int_0^\infty \frac{dx}{1+x^4}$ in terms of Beta function and hence evaluate. (06 Marks)

4 a. Select correct answer in each of the following:

i) If $\vec{F}$ is the force acting upon a particle moves from one end of a curve C to the other end then the total work done by $\vec{F}$ is.

A) $\int \vec{F}\times d\vec{r}$ B) $\int \vec{F}\cdot d\vec{r}$ C) $d\vec{r}$ D) None of these

ii) The line integral of $\vec{F}=x^2i+xyj$ from O(0, 0) to P(1, 1) along the straight path is.

A) $\frac{1}{3}$ B) $\frac{5}{3}$ C) $\frac{2}{3}$ D) $\frac{4}{3}$

iii) If M, N, $\frac{\partial N}{\partial x}$, $\frac{\partial M}{\partial y}$ are continuous functions. C is a simple closed curve enclosing the region R in the xy plane then Green's theorem states that.

A) $\oint_C Mdx+Ndy=\iint_R\left(\frac{\partial N}{\partial x}-\frac{\partial M}{\partial y}\right)dxdy$ B) $\oint_C Mdx+Ndy=\iint_R\left(\frac{\partial N}{\partial x}+\frac{\partial M}{\partial y}\right)dxdy$

C) $\oint_C Mdx+Ndy=\iint_R\left(\frac{\partial M}{\partial y}-\frac{\partial N}{\partial x}\right)dxdy$ D) $\oint_C Mdx-Ndy=\iint_R\left(\frac{\partial M}{\partial x}-\frac{\partial N}{\partial y}\right)dxdy$

iv) The cylindrical coordinate system is.

A) Not orthogonal B) Orthogonal C) Coplanar D) Non-coplanar (04 Marks)

b. Find the total work done by the force represented by $\vec{F}=3xyi-yj+2xzk$ in a moving particle round the circle $x^2+y^2=4$. (04 Marks)

c. Verify Stokes theorem for the vector field, $\vec{F}=(2x-y)i-yz^2j-y^2zk$ over the upper half surface of $x^2+y^2+z^2=1$ bounded by its projection on the xy-plane. (06 Marks)

d. Express the vector $\vec{F}=zi-2xj+yk$ in cylindrical coordinates. (06 Marks)

Part B

5 a. Select the correct answer in each of the following :

i) The solution of the different equation $(D^2 - a^2)y = 0$ is
A) $C_1e^{ax} + C_2e^{-ax}$ B) $(C_1 + C_2x)e^{ax}$ C) $(C_1 + C_2x^2)e^{ax}$ D) $(C_1x+C_2x^2)e^{ax}$

ii) P.I. of the differential equation $(D^2 + 5D + 6)y = ex$ is
A) e^x B) $\frac{e^x}{12}$ C) $12e^x$ D) $\frac{e^x}{6}$

iii) C.F. of $y'' - 2y' + y = x e^x \sin x$ is
A) $C_1e^x + C_2e^{-x}$ B) $(C_1 + C_2x) e^x$ C) $C_1 + C_2e^x$ D) None of these

iv) P.I. of $y'' - 3y' + 2y = 4$ is
A) 2 B) $\frac{1}{2}$ C) $\frac{1}{4}$ D) $\frac{3}{2}$ (04 Marks)

b. Solve $\frac{d^3y}{dx^3} + 6\frac{d^2y}{dx^2} + 11\frac{dy}{dx} + 6y = e^{2x}$. (04 Marks)

c. Solve the equation $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 5y = e^{2x}\sin x$. (06 Marks)

d. Using the method of undetermined coefficient solve the equation,
$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 4y = 2x^2 + 3e^{-x}$. (06 Marks)

6 a. Select the correct answer in each of the following :

i) The Wronskian of cos x and sin x is
A) 0 B) 1 C) 2 D) 4

ii) To transform $x^2\frac{d^2y}{dx^2} - x\frac{dy}{dx} + y = \log x$ into a linear differential equation with constant coefficients put x =
A) e^t B) Log x C) e^x D) t

iii) The solution of the differential equation $y'' + y = 0$, satisfying the conditions $y(0) = 1$ and $y\left(\frac{\pi}{2}\right) = 2$ is
A) Cos x + 2 sin x B) 2Cos x + Sin x C) Cos x − Sin x D) None of these

iv) $C_1 \cos ax + C_2 \sin ax - \frac{x}{2a}\cos ax$ is the general solution of
A) $(D^2 + a^2)y = \sin ax$ C) $(D^2 + a^2)y = \cos ax$
B) $(D^2 - a^2)y = \sin ax$ D) $(D + a)y = \sin x$. (04 Marks)

b. Solve $x^2\frac{d^2y}{dx^2} - x\frac{dy}{dx} + y = \log x$. (04 Marks)

c. Solve $y''-2y'+y=e^x \log x$ by using method of variation of parameters. (06 Marks)

d. Solve $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 5y = 0$, given that y = 2 and $\frac{dy}{dx} = \frac{d^2y}{dx^2}$ when x = 0. (06 Marks)

7 a. Select the correct answer in each of the following

i) Laplace transform of f(t) : $t \ge 0$ is defined as

A) $\int_0^\infty e^{-st} f(t)dt$ B) $\int_0^1 e^{-st} f(t)dt$ C) $\int_1^\infty e^{-st} f(t)dt$ D) $\int_0^\infty e^{st} f(t)dt$

ii) Laplace transform of sin at is

A) $\frac{1}{S^2+a^2}$ B) $\frac{a}{S^2+a^2}$ C) $\frac{S}{S^2+a^2}$ D) $\frac{1}{S^2-a^2}$

iii) Laplace transform of $e^{3t}\sin t$ is

A) $\frac{1}{S^2+S+10}$ B) $\frac{1}{S^2-6S+10}$ C) $\frac{1}{S^2+4S+10}$ D) $\frac{S}{S^2-6S+10}$

iv) Laplace transform of $f'(t) =$

A) $s\,f(s)-f(0)$ B) $s\,f'(s)\ f(0)$ C) $f(s)-f(0)$ D) $s\,f'(s)-f(0)$.

(04 Marks)

b. Find the Laplace transform of $e^{2t}\cos^2 t$ (04 Marks)

c. Find the Laplace transform of the periodic function of period 4 defined by,

$$f(t) = \begin{cases} t & \text{for } 0<t<2 \\ 0 & \text{for } 2<t<4 \end{cases}$$

(06 Marks)

d. Express the function $f(t) = \begin{cases} \pi - t & \text{for } 0<t<\pi \\ \sin t & \text{for } t>\pi \end{cases}$

in terms of unit step function and find its Laplace transform. (06 Marks)

8 a. Select the correct answer in each of the following :

i) Inverse Laplace transform of $\frac{s-a}{(s-a)^2+b^2}$ is

A) $e^t\cos bt$ B) $e^{at}\cos bt$ C) $e^{-at}\cos bt$ D) $e^{at}\sin bt$.

ii) Inverse Laplace transform of $\frac{s^2-3s+4}{s^3}$ is

A) $1-3t+2t^2$ B) $10-3t+2t^2$ C) $4-3t+4t^2$ D) $5-6t+2t^2$.

iii) $L^{-1}\left\{\frac{1}{s^n}\right\}$ is possible only when n is

A) negative integer B) positive integer C) zero D) Real number

iv) $L^{-1}\left\{\frac{1}{(s-3)^5}\right\} =$

A) $\frac{e^{3t}t^4}{24}$ B) $\frac{e^{5t}t^4}{24}$ C) $\frac{e^{2t}t^4}{24}$ D) $\frac{e^{-3t}t^3}{24}$ (04 Marks)

b. Find the inverse Laplace transform of $\frac{3s+2}{s^2-s-2}$. (04 Marks)

c. Using the convolution theorem obtain the inverse Laplace transform of $\frac{s^2}{(s^2+a^2)(s^2+b^2)}$.

(06 Marks)

d. Solve the differential equation $\frac{d^2y}{dt^2}+4\frac{dy}{dt}+4y = e^{-t}$ with $y(0) = 0 = y'(0)$, using Laplace transform method. (06 Marks)

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06MAT21

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Second Semester B.E. Degree Examination, July 2007
Common to All Branches
Engineering Mathematics – II

Time: 3 hrs.] [Max. Marks:100

Note : ***Answer any FIVE full questions choosing at least TWO full questions from each part.***

Part A

1 a. Derive the expression for radius of curvature in polar coordinates. **(06 Marks)**

b. Employ Lagrange's mean value theorem, prove that

$$\frac{b-a}{\sqrt{1-a^2}} < \sin^{-1}(b) - \sin^{-1}(a) < \frac{b-a}{\sqrt{1-b^2}}$$

where a < b < 1 and deduce that $\frac{\pi}{6} + \frac{1}{5\sqrt{3}} < \sin^{-1}\left(\frac{3}{5}\right) < \frac{\pi}{6} + \frac{1}{8}$. **(07 Marks)**

c. Prove that $e^{x\cos x} = 1 + x + \frac{x^2}{\angle 2} - 2\frac{x^3}{\angle 3} - 11\frac{x^4}{\angle 4} + \ldots\ldots\ldots$ **(07 Marks)**

2 a. Evaluate $\underset{x\to 0}{\text{limit}}\left[\frac{a^x + b^x + c^x + d^x}{4}\right]^{1/x}$ **(06 Marks)**

b. Expand $xy^2 + \cos(xy)$ in the powers of x and y upto third degree terms about $(1, \pi/2)$. **(07 Marks)**

c. Use Lagrange's multipliers method to find the maximum of the function $x^2y^2z^2$ subject to the condition $x^2 + y^2 + z^2 = a^2$. **(07 Marks)**

3 a. Change the order of integration in $\int_0^1 \int_{x^2}^{2-x} xy\, dy\, dx$ and hence evaluate the same. **(06 Marks)**

b. Evaluate $\iiint (x+y+z)dx\, dy\, dz$ over the tetrahedron bounded by the planes x = 0, y = 0, z = 0 and x + y + z = 1 **(07 Marks)**

c. Express $\int_0^1 x^m (1-x^n)^p dx$ interms of Gamma functions and evaluate $\int_0^1 x^5 (1-x^3)^{10} dx$. **(07 Marks)**

4 a. Verify Green's theorem for $\int_c \left[(xy + y^2)dx + x^2 dy\right]$, where c is bounded by y = x and $y = x^2$. **(06 Marks)**

b. Apply Stoke's theorem to evaluate $\int_c [ydx + zdy + xdz]$ where c is the curve of intersection of $x^2 + y^2 + z^2 = a^2$ and x + z = a. **(07 Marks)**

c. Prove that the cylindrical coordinate system is orthogonal. **(07 Marks)**

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Part B

5 a. Solve $(D^4 + 2D^3 - 5D^2 - 6D)y = 0$. (06 Marks)

b. Solve $(D-2)^2 y = 8(e^{2x} + \sin 2x + x^2)$. (07 Marks)

c. Solve $D^2(D-1)y = 3e^x + \sin x$ by the method of undetermined coefficients. (07 Marks)

6 a. Solve $(D^2 - 3D + 2)y = \dfrac{1}{1+e^{-x}}$ by the method of variation of parameters. (06 Marks)

b. Solve $(1+x)^2 y'' + (1+x)y' + y = 2\sin\log(1+x)$. (07 Marks)

c. Solve $y'' + 4y' + 5y = -2\cosh x$, also find y when $y = 0,\ y' = 1$ at $x = 0$. (07 Marks)

7 a. Prove that

i) $L\{t^n\} = \dfrac{\angle n}{S^{n+1}}$

ii) $L\{t^n f(t)\} = (-1)^n \dfrac{d^n[F(S)]}{ds^n}$ (06 Marks)

b. Find the Laplace transform of the triangular wave of period 2a given by

$$f(t) = \begin{cases} t, & 0 < t < a \\ 2a - t, & a < t < 2a \end{cases}$$ (07 Marks)

c. Define unit step function and the transform of unit step function and find the Laplace transform of

$$f(t) = \begin{cases} t-1, & 1 < t < 2 \\ -(t+3), & 2 < t < 3 \end{cases}$$ (07 Marks)

8 a. Find $L^{-1}\left\{\dfrac{2S^2 - 6S + 5}{S^3 - 6S^2 + 11S - 6}\right\}$. (06 Marks)

b. Find $L^{-1}\left\{\dfrac{1}{S^3(S^2+1)}\right\}$ using convolution theorem. (07 Marks)

c. Solve $y'' - 3y' + 2y = 4t + e^{3t}$ with $y(0) = 1,\ y'(0) = -1$ using Laplace transform. (07 Marks)

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06MAT21

Second Semester B.E. Degree Examination, Dec. 07 / Jan. 08
Engineering Mathematics – II

Time: 3 hrs. Max. Marks:100

Note : ***Answer any FIVE questions choosing atleast TWO questions from each part.***

PART – A

1 a. For the curve $y = \frac{ax}{a+x}$, show that $\left(\frac{2\rho}{a}\right)^{2/3} = \left(\frac{x}{y}\right)^2 + \left(\frac{y}{x}\right)^2$. **(06 Marks)**

b. State and prove Cauchy's mean value theorem. **(07 Marks)**

c. Expand $e^{\sin x}$ by Maclaurin's series upto the term containing x^4. **(07 Marks)**

2 a. Evaluate

i) $\lim_{x \to 0} \frac{a^x - b^x}{x}$

ii) $\lim_{x \to \infty} \left(\frac{1^{\frac{1}{x}} + 2^{\frac{1}{x}} + 3^{\frac{1}{x}}}{3} \right)^{3x}$ **(06 Marks)**

b. Expand $\tan^{-1}\left(\frac{y}{x}\right)$ about the point (1, 1) up to 2^{nd} degree terms using Taylor's series. **(07 Marks)**

c. Find the minimum value of $x^2 + y^2 + z^2$ when $x + y + z = 3a$. **(07 Marks)**

3 a. Evaluate the integral $\int_0^a \int_y^a \frac{x}{x^2 + y^2} dxdy$ by changing the order of integration. **(06 Marks)**

b. Evaluate the integral $\int_0^1 \int_0^{\sqrt{1-x^2}} \int_{\sqrt{x^2+y^2}}^{0} \frac{dzdydx}{\sqrt{x^2 + y^2 + z^2}}$. **(07 Marks)**

c. Show that $\beta(m,n) = \int_0^1 \frac{\left(x^{m-1} + x^{n-1}\right)}{(1+x)^{m+n}} dx$. **(07 Marks)**

4 a. Using Green's theorem in the plane evaluate $\int_C \left[\left(2x^2 - y^2\right)dx + \left(x^2 + y^2\right)dy\right]$ where C is the boundary of the region bounded by $x = 0$, $y = 0$, $x + y = 1$. **(06 Marks)**

b. Using Divergence theorem evaluate $\iint_s \vec{F} \cdot \hat{n}\, ds$ where $\vec{F} = (2x + 3z)\hat{i} - (xz + y)\hat{j} + \left(y^2 + 2z\right)\hat{k}$ and s is the surface of the sphere having center at (3, –1, 2) and radius 3. **(07 Marks)**

c. Prove that spherical polar coordinate system is orthogonal. **(07 Marks)**

PART – B

5 a. Solve : $(4D^4 - 8D^3 - 7D^2 + 11D + 6)y = 0$ (06 Marks)

b. Solve : $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = xe^{3x} + \sin 2x$. (07 Marks)

c. Using the method of undetermined coefficients solve $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 4y = 2x^2 + 3e^{-x}$. (07 Marks)

6 a. Solve : $\frac{d^2y}{dx^2} + \frac{dy}{dx} - 2y = \frac{1}{1-e^x}$ by using the method of variation of parameters. (06 Marks)

b. Solve : $(2x+3)^2\frac{d^2y}{dx^2} + 5(2x+3)\frac{dy}{dx} + y = 4x$. (07 Marks)

c. Solve : $\frac{d^2x}{dt^2} - 4\frac{dx}{dt} + 13x = 0$ with $x(0) = 0$, $\frac{dx(0)}{dt} = 2$. (07 Marks)

7 a. Evaluate :

i) $L\{te^{-2t}\sin 4t\}$ ii) $L\left\{\frac{1-\cos at}{t}\right\}$ (06 Marks)

b. Define periodic function. If f(t) is a periodic function with period T then show that

$$L\{f(t)\} = \frac{1}{1-e^{-st}}\int_0^T e^{-st}f(t).dt$$ (07 Marks)

c. Express $f(t) = \begin{cases} 1 & \text{if} \quad 0 < t \le 1 \\ t & \text{if} \quad 1 < t \le 2 \\ t^2 & \text{if} \quad t > 2 \end{cases}$ in terms of unit step function and hence find $L\{f(t)\}$. (07 Marks)

8 a. Find :

i) $L^{-1}\left\{\frac{s+2}{s^2-4s+13}\right\}$ ii) $L^{-1}\left\{\log\frac{s^2+1}{s(s+1)}\right\}$ (06 Marks)

b. Find the inverse Laplace transform of $\frac{s}{(s^2+1)(s^2+4)}$ using convolution theorem. (07 Marks)

c. Solve the differential equation

$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = e^{2x}$, $y(0) = 2$, $y'(0) = -1$ by using Laplace transforms. (07 Marks)

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06MAT21

Second Semester B.E. Degree Examination, June/July 08

Engineering Mathematics II

Time: 3 hrs. Max. Marks:100

Note : ***Answer any FIVE full questions, choosing at least two full questions from each part.***

Part - A

1 a. Find the radius of curvature of the curve
$x^3 + y^3 = 3axy$ at the point $\left(\frac{3a}{2}, \frac{3a}{2}\right)$ **(06 Marks)**

b. State and prove Lagrange's mean value theorem. **(07 Marks)**

c. Expand $e^{\tan^{-1} x}$ by Maclaurin's series upto the term containing x^5 **(07 Marks)**

2 a. Evaluate :
i) $\lim_{x \to \pi/2} (2x \tan x - \pi \sec x)$

ii) $\lim_{x \to a} \left[2 - \left(x/a\right)\right]^{\tan\left(\frac{\pi x}{2a}\right)}$ **(06 Marks)**

b. Expand $x^2y + 3y - 2$ in powers of (x - 1) and (y + 2) using the Taylor's theorem. **(07 Marks)**

c. Find the maximum and minimum values of $x^2 + y^2$ subject to the condition $5x^2 + 6xy + 5y^2 = 8$ **(07 Marks)**

3 a. Evaluate the integral $\int_0^1 \int_{x^2}^{2-x} xy \, dy \, dx$ by changing the order of integration. **(07 Marks)**

b. Evaluate the integral $\int_0^1 \int_0^{1-x} \int_0^{1-x-y} \frac{dz \, dy \, dx}{(1+x+y+z)^3}$ **(07 Marks)**

c. With the usual notation, show that
$\beta(m,n) = \frac{\Gamma(m) \cdot \Gamma(n)}{\Gamma(m+n)}$ **(06 Marks)**

4 a. Verify Green's theorem for $\oint_c \left[(xy + y^2)dx + x^2 dy\right]$ where c is the closed curve of the region bounded by y = x and $y = x^2$. **(07 Marks)**

b. Using the divergence theorem evaluate
$\iint_s \vec{f} \cdot \hat{n} ds$ where $\vec{f} = x^3\hat{i} + y^3\hat{j} + z^3\hat{k}$ and s is the surface of the sphere $x^2 + y^2 + z^2 = a^2$ **(07 Marks)**

c. Prove that cylindrical coordinate system is orthogonal. **(06 Marks)**

Part - B

5 a. Solve: $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = e^{2x} + \cos 2x$ **(07 Marks)**

b. Solve: $\frac{d^3y}{dx^3} + y = 5e^x x^2$ **(07 Marks)**

c. Using the method of undetermined coefficients
Solve: $\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = e^{3x}$ **(06 Marks)**

II. H2

6 a. Solve: $\frac{d^2y}{dx^2}+4y=\tan 2x$ by using the method of variation of parameters. (07 Marks)

b. Solve: $x^3\frac{d^3y}{dx^3}+3y^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}+8y=65\ \cos(\log x)$ (07 Marks)

c. Solve the initial value problem

$$\frac{d^2x}{dt^2}+5\frac{dx}{dt}+6x=0, \text{given that } x(0)=0, \frac{dx(0)}{dt}=15.$$ (06 Marks)

7 a. Find the Laplace transforms of

i) t^2e^{2t}

ii) $(\cos at-\cos bt)/t$ (07 Marks)

b. Find Laplace transform of the periodic function of period 2 a, which is defined by

$$f(t)=\begin{cases} t & \text{if } o\le t\le a \\ 2a-t & \text{if } a\le t\le 2a \end{cases}$$ (07 Marks)

c. Express $f(t)=\begin{cases} \cos t, & 0<t<\pi \\ \cos 2t, & \pi<t<2\pi \\ \cos 3t, & t>2\pi \end{cases}$ (06 Marks)

In terms of Heaviside unit step function and hence find $L\{f(t)\}$

8 a. Find i) $L^{-1}\left\{\frac{s-2}{s^2+7s+12}\right\}$

ii) $L^{-1}=\left\{\frac{e^{-6s}}{(s-4)^2}\right\}$ (06 Marks)

b. Using convolution theorem obtain the inverse Laplace transform of $\frac{s}{(s+2)(s^2+9)}$. (07 Marks)

c. Solve the differential equation $\frac{d^2y}{dx^2}+4\frac{dy}{dx}+3y=e^{-x}$ with $y(0)=1=y'(0)$ using Laplace transforms. (07 Marks)

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2nd Sem. VTU

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06MAT21

Second Semester B.E. Degree Examination, Dec.08 / Jan.09
Engineering Mathematics - II

Time: 3 hrs. Max. Marks:100

Note : ***Answer FIVE full questions selecting at least two from each part.***

PART – A

1 a. Find the radius of curvature at the point $\left(\frac{3a}{2},\frac{3a}{2}\right)$ of the Folium $x^3+y^3=3axy$. **(06 Marks)**

b. State and prove Cauchy's mean value theorem. **(07 Marks)**

c. If $f(x)=\log(1+x)$, $x>0$, using Maclaurin's theorem, show that $\log(1+x)=x-\frac{x^2}{2}+\frac{x^3}{3(1+\theta x)^3}$, for $0<\theta<1$. Deduce that $\log(1+x)<x-\frac{x^2}{2}+\frac{x^3}{3}$, for $x>0$. **(07 Marks)**

2 a. Evaluate i) $\lim_{x\to 0}(\cos x)^{1/x^2}$.

ii) $\lim_{x\to 0}\left(\frac{1}{\sin x}-\frac{1}{x}\right)$. **(06 Marks)**

b. Expand $f(x,y)=\tan^{-1}\left(y/x\right)$ in powers of $(x-1)$ and $(y-1)$ up to second degree terms. Hence compute $f(1.1,0.9)$ approximately. **(07 Marks)**

c. Discuss the maxima and minima of $f(x,y)=x^3y^2(1-x-y)$. **(07 Marks)**

3 a. Evaluate the integral by changing the order of integration,

$\int_0^\infty\int_0^x xe^{-x^2/y}\,dydx$. **(06 Marks)**

b. Find the volume of the sphere $x^2+y^2+z^2=a^2$ using triple integration. **(07 Marks)**

c. Express the following integrals in terms of Gamma functions:

i) $\int_0^1\frac{dx}{\sqrt{1-x^4}}$

ii) $\int_0^\infty\frac{x^C}{C^x}dx$ **(07 Marks)**

4 a. Find the work done in moving a particle in the force field $\vec{F}=3x^2i+(2xz-y)j+zk$ along the straight line from (0, 0, 0) to (2, 1, 3). **(06 Marks)**

b. Verify Green's theorem for $\int_C\left[(xy+y^2)dx+x^2dy\right]$, where C is bounded by $y=x$ and $y=x^2$. **(07 Marks)**

c. Prove that the cylindrical coordinate system is orthogonal. **(07 Marks)**

PART – B

5 a. Solve $\frac{d^3y}{dx^3}+2\frac{d^2y}{dx^2}+\frac{dy}{dx}=e^{-x}+\sin 2x$. **(06 Marks)**

b. Solve $\frac{d^2y}{dx^2}-4y=\cosh(2x-1)+3^x$. **(07 Marks)**

c. Solve by the method of undetermined coefficients, $(D^2+1)y=\sin x$. **(07 Marks)**

6 a. Solve $(1+x)^2y''+(1+x)y'+y=2\sin[\log(1+x)]$. **(06 Marks)**

b. Solve $x^3\frac{d^3y}{dx^3}+2x^2\frac{d^2y}{dx^2}+2y=10\left(x+\frac{1}{x}\right)$. **(07 Marks)**

c. Solve, by the method of variation of parameters $\frac{d^2y}{dx^2}+y=\tan x$. **(07 Marks)**

7 a. Find the Laplace transforms of

i) $2^t+\frac{\cos 2t-\cos 3t}{t}$.

ii) $\int_0^t \frac{\sin t}{t}dt$. **(06 Marks)**

b. Find the Laplace transform of the periodic function with period $\frac{2\pi}{\omega}$.

$$f(t)=\begin{cases}\sin\omega t, & 0<t<\frac{\pi}{\omega}\\ 0, & \frac{\pi}{\omega}<t<\frac{2\pi}{\omega}\end{cases}$$ **(07 Marks)**

c. Express $f(t)=\begin{cases}\cos t, & 0<t<\pi\\ \cos 2t, & \pi<t<2\pi\\ \cos 3t, & t>2\pi\end{cases}$

in terms of unit step function and hence find its Laplace transform. **(07 Marks)**

8 a. Find i) $L^{-1}\left[\frac{s+3}{s^2-4s+13}\right]$.

ii) $L^{-1}\left[\log\left(\frac{s+a}{s+b}\right)\right]$. **(06 Marks)**

b. Apply convolution theorem to evaluate $L^{-1}\left[\frac{s}{(s^2+a^2)^2}\right]$. **(07 Marks)**

c. Solve the differential equation by Laplace transform method,
$y''+4y'+3y=e^{-t}$ and the initial conditions $y(0)=y'(0)=1$. **(07 Marks)**

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06MAT21

Second Semester B.E. Degree Examination, June–July 2009
Engineering Mathematics - II

Time: 3 hrs. Max. Marks:100

Note : 1. ***Answer any Five full question, choosing at least two from each part.***
2. ***Answer all objectives type questions only in OMR sheet page 5 of the Answer Booklet.***
3. ***Answer to the objective type questions on sheets other than OMR will not be valued***

PART – A

1 a. Select correct answer in each of the following :
i) Curvature of a circle is
A) a constant B) a variable C) a straight line D) none of these.
ii) Radius of curvarture for the Cartesian curve y = f(x) is
A) $\frac{(1+y_2^2)^{3/2}}{y_2}$ B) $\frac{(1+y_1^2)^{3/2}}{y_2}$ C) $\frac{(1+y_1^2)^3}{y_2}$ D) $\frac{(1+y_1^2)^2}{y_2}$
iii) If f(x) is continuous in the closed interval [a,b] , differentiable in (a,b) and f(a) = f(b) then there exists at least one value c of x in (a,b) such that f ' (c) is equal to
A) 1 B) -1 C) 2 D) 0
iv) Maclaurin's series expansion of e^x is
A) $1+x+\frac{x^2}{2!}+\frac{x^3}{3!}+....$ B) $x-\frac{x^3}{3!}+\frac{x^5}{5!}-.....$
C) $x+\frac{x^3}{3}+\frac{x^5}{5}+.....$ D) $1-\frac{x^2}{2!}+\frac{x^4}{4!}-.....$ **(04 Marks)**

b. Show that the radius of curvature of the curve y = 4 Sin x – Sin 2x at $x=\frac{\pi}{2}$ is $\frac{5\sqrt{5}}{4}$. **(04 Marks)**

c. Verify Lagrange's mean value theorem for the function f(x) = (x-1) (x-2) (x-3) in [0,4]. **(06 Marks)**

d. Expand log(secx) using Maclaurin's series upto the term containing x^4. **(06 Marks)**

2 a. Select correct answer in each of the following :
i) $\underset{x\to 1}{\text{Lt}}\ \frac{x^2-4x+3}{x^2-3x+2}$ is equal to
A) 0 B) -2 C) 2 D) -1
ii) f(a,b) is said to be a stationary value of f(x,y) if
A) $f_x(a,b) = 0$, $f_y(a,b) \neq 0$ B) $f_x(a,b) = 0$, $f_y(a,b) = 0$
C) $f_{xx}(a,b) = 0$, $f_{yy}(a,b) = 0$ D) $f_{xy}(a,b) = 0$, $f_{yy}(a,b) = 0$.
iii) If $r = f_{xx}(a,b)$, $s = f_{xy}(a,b)$, $t = f_{yy}(a,b)$ then f(x,y) will have a minimum at (a,b) if
A) $f_x = 0$, $f_y = 0$ $rt - s^2 > 0$ and $r > 0$ B) $f_x = 0$, $f_y = 0$. $rt - s^2 > 0$ and $r < 0$
C) $f_x = 0$, $f_y = 0$ $rt - s^2 = 0$ and $r > 0$ D) $f_x = 0$, $f_y = 0$, $rt - s^2 > 0$ and $r = 0$.

iv) The volume of the greatest rectangular parallelopiped that can be inscribed in the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ is

A) $\frac{16abc}{3\sqrt{3}}$ B) $\frac{8abc}{3\sqrt{3}}$ C) $\frac{24abc}{3\sqrt{3}}$ D) $\frac{4abc}{3\sqrt{3}}$ **(04 Marks)**

b. Evaluate $\underset{x\to 0}{\text{Lt}}$ tan x log x. **(04 Marks)**

c. Expand f(x,y) = Sinx cosy in powers of x and y as far as the terms of third degree. **(06 Marks)**

d. The temperature T at any point (x,y,z) in space is $T = 400\,xyz^2$. Find the highest temperature on the surface of the unit sphere $x^2 + y^2 + z^2 = 1$. **(06 Marks)**

3 a. Select correct answer in each of the following : **(04 Marks)**

i) $\int_b^a \int_0^b dxdy =$

A) 0 B) $\frac{ab}{2}$ C) 2ab D) ab.

ii) Volume of a solid is equal to

A) ∭ dx dy dz B) ∬ dx dy C) ∬ xy dx dy D) None of these.

iii) The value of $\int_0^1 x^7(1-x)^8 dx$ is

A) β(7,8) B) β(8,9) C) β(7,9) D) None of these

iv) The value of Γ(n + 1) is

A) (n+1)! B) (n + 1) Γ (n + 1) C) n Γ(n) D) (n - 1) Γ (n -1)

b. Evaluate ∬ xy(x + y) dx dy taken over the region enclosed by the curves y = x and $y = x^2$. **(04 Marks)**

c. Evaluate $\int_0^{\pi/2} \int_0^{(a\sin\theta)} \int_0^{\left(\frac{a^2-r^2}{a}\right)} r\,dr\,d\theta\,dz$. **(06 Marks)**

d. Evaluate $\int_0^{\pi/2} \sqrt{\cot\theta}\, d\theta$ by expressing interms of gamma functions. **(06 Marks)**

4 a. Select correct answer in each of the following :

i) If $\vec{F}$ is irrotational around every closed curve C, then.

A) $\int_c \vec{F}.d\vec{r} = 0$ B) $\int_c \vec{F}\times d\vec{r} = 0$ C) $\int_c d\vec{r} = 0$ D) None of these

ii) If $\vec{F} = x^2\,i + xy\,j$ then the value of $\int \vec{F}.d\vec{r}$ from (0,0) to (1,1) along the line y = x is

A) $\frac{3}{2}$ B) $\frac{2}{3}$ C) 3 D) 2

iii) Green's theorem in the plane is a special case of

A) Gauss theorem B) Euler's theorem C) Stokes theorem D) Baye's theorem.

iv) The spherical coordinate system is

A) Orthogonal B) Coplanar C) Collinear D) Not orthogonal **(04 Marks)**

b. Using Green's theorem in the plane, evaluate $\int\{(2x^2 - y^2)dx + (x^2 + y^2)dy\}$, where C is the boundary of the region bounded by x = 0, y = 0, x + y = 1. **(04 Marks)**

c. Apply Stoke's theorem, to evaluate $\int_c \vec{F}.d\vec{r}$ where

$\vec{F} = (y + z - 2x)\, i + (z + x - 2y)\, j + (x + y - 2z)\, k$ and c is the triangle with vertices (1,0,0), (0,2,0) and (0,0,3). **(06 Marks)**

d. Express the vector $\vec{F} = 2\,x\,i - 3\,y^2\,j + x\,z\,k$, in cylindrical polar coordinate system. **(06 Marks)**

PART – B

5 a. Select correct answer in each of the following :

i) Given f(D) y = x and if m_1, m_2 are real and distinct roots of the A – E then C – F is

A) $(C_1 + C_2\,x)\,e^{(m_1+m_2)x}$ B) $C_1 \cos m_1x + C_2 \sin m_2\,x$

C) $(C_1 + C_2)e^{mx}$ C) $C_1e^{m_1x} + C_2e^{m_2x}$

ii) P.I. of the differential equation $(D^2 - 7D + 12)\,y = e^{2x}$ is

A) $2\,e^{2x}$ B) $\frac{e^{2x}}{2}$ C) $4e^{2x}$ D) $\frac{e^{2x}}{4}$

iii) The solution of the differential equation $(D^2 - 6D + 13)\,y = 0$ is

A) $3 \pm 2i$ B) $C_1\,e^{3x} + C_2\,e^{2x}$ C) $e^{3x}(C_1 \cos 2x + C_2\sin 2x)$ D) $(C_1 + C_2x)\,e^{3x}$

iv) By the method of undetermined coefficients P.I. of $y'' + 2y' + 4y = 2x^2 + 3e^{-x}$ will be of the form.

A) $a + bx + cx^2$ B) $(a + bx + cx^2) + de^{-x}$ C) $ax^2 + be^{-x}$ D) $a \cos x + b \sin x$. **(04 Marks)**

b. Find the P.I. of $(D^2 - 5D + 1)y = 1 + x^2$. **(04 Marks)**

c. Solve the equation $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 5y = e^{2x}\sin x$. **(06 Marks)**

d. Using method of undetermined coefficients, solve the equation $y'' + 2y' + 3y = x^2 - \cos x$. **(06 Marks)**

6 a. Select correct answer in each of the following :

i) To find the P.I of the equation $f(D)y = e^{ax}$, by the method of undetermined coefficients, we assume a trail solution as

A) $\frac{1}{f(D)}e^{ax}$ B) Ae^{ax} C) $\frac{1}{f(a)}e^{ax}$ D) None of these

ii) The C.F of $x^2y'' + xy' + y = 2\cos^2(\log x)$ is

A) $C_1 \cos(\log x) + C_2 \sin(\log x)$ B) $C_1 \cos x + C_2 \sin x$

C) $C_1 \log(\cos x) + C_2 \log(\sin x)$ D) None of these.

iii) Cauchy's differential equation is a special case of Legendre's linear equation if

A) a = 1, b = 1 B) a = 0, b = 1 C) a = 1, b = 0 D) a = 2, b = 2

iv) A differential equation y'' - y' = 0 with the conditions y(0) = 1, y(1) = 2 – e constitute

A) a initial value problem B) a boundary value problem

C) a probability value problem C) a bending value problem. **(04 Marks)**

b. Solve the initial value problem $\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 6x = 0$ given that $x(0) = 0$, $\frac{dx}{dt}(0) = 15$. **(04 Marks)**

c. Solve $(1+x)^2\frac{d^2y}{dx^2} + (1+x)\frac{dy}{dx} + y = 2\sin[\log(1+x)]$. **(06 Marks)**

d. Solve the equation y'' + y = tan x, by the method of variation of parameters. **(06 Marks)**

7 a. Select correct answer in each of the following :

i) Laplace transform of f(t) : $t \geq 0$ is defined as

A) $\int_0^\infty e^{st}f(t)dt$ B) $\int_0^1 e^{-st}f(t)dt$ C) $\int_1^\infty e^{-st}f(t)dt$ D) $\int_0^\infty e^{-st}f(t)dt$

ii) Laplace transform of e^{-at} is

A) $\frac{1}{s-a}$ B) $\frac{1}{s+a}$ C) $\frac{1}{s^2+a^2}$ D) $\frac{1}{s^2-a^2}$

iii) Laplace transform of $\sin 2t\delta(t-2)$ is

A) $e^{2s}\sin 4$ B) $e^{-2s}\sin 2$ C) $e^{-4s}\sin 2$ D) $e^{-2s}\sin 4$.

iv) $L\left\{ \int_0^t f(t)dt \right\} =$

A) $\frac{1}{t}$ L {f(t)} B) $\frac{1}{s}$ L {f(t)} C) $\frac{1}{s^2}$ L {f(t)} D) None of these. **(04 Marks)**

b. Find the Laplace transform of $\cos 3t + 2^t$. **(04 Marks)**

c. Find the Laplace transform of $\frac{e^{-t}\sin t}{t}$ and hence deduce that $\int_0^\infty \frac{e^{-t}\sin t}{t}dt = \frac{\pi}{4}$ **(06Marks)**

d. Find the Laplace transform of the square wave function of period 2a define by
f(t) = k when $0 < t < a$
= - k when $a < t < 2a$. **(06 Marks)**

8 a. Select correct answer in each of the following :

i) $L^{-1}\left\{\frac{1}{s}\right\} =$

A) 0 B) 1 C) $\frac{1}{2}$ D) 2

ii) Inverse Laplace transform of $\frac{1}{(s-a)^2+b^2}$ is

A) $\frac{1}{b}e^{at}\sin bt$ B) $e^{at}\sin bt$ C) $\frac{1}{a}e^{at}\sin bt$ D) None of these.

iii) $L^{-1}\left\{\frac{1}{(s-1)^2}\right\} =$

A) te^t B) e^t C) $\frac{1}{2}e^t$ D) e^{t-1}

iv) $L^{-1}\{e^{-as}F(s)\} =$

A) f(t) u(t) B) f(t-a) u(t) C) f(t-a) u(t-a) D) None of these. **(04 Marks)**

b. Find the inverse Laplace transform of $\frac{3s+7}{s^2-2s-3}$. **(04 Marks)**

c. Using Convolution theorem, find the inverse Laplace transform of $\frac{s}{(s^2+a^2)^2}$. **(06 Marks)**

d. Applying Laplace transform method, solve the equation
$y'' + 5y' + 6y = 5e^{2x}$, given $y(0) = 2$, $y'(0) = 1$. **(06 Marks)**

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06MAT21

Second Semester B.E. Degree Examination, Dec.09-Jan.10
Engineering Mathematics - II

Time: 3 hrs. Max. Marks:100

Note:1. ***Answer any FIVE full questions, selecting atleast TWO questions from each Part.***
2. ***Answer all objectives type questions only on OMR sheet, page No 5 of the Answer booklet.***
3. ***Answer to the objective type questions on sheets other than OMR will not be valued.***

PART - A

1 a. Select correct answer in each of the following : **(04 Marks)**

i) The radius of curvature at a point (x,y) of $y = a\cosh\left(\frac{x}{a}\right)$ is,

A) $\frac{y^2}{a}$ B) $\frac{x^2}{a}$ C) $\frac{a^2}{y}$ D) None of these

ii) The radius of the circle of curvature is,

A) 1 B) ρ C) $\frac{1}{\rho}$ D) ρ^2.

iii) The Lagrange's mean value theorem for the function $f(x) = e^x$ in the interval [0,1] is,

A) C = 0.5413 B) C = 2.3 C) C = 0.3 D) None of these

iv) Maclaurin's expansion of e^x is

A) $1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots$ B) $1 - \frac{x^2}{2!} + \frac{x^3}{3!} - \ldots$

C) $x + x^2 + x^3 + \ldots$ D) None of these

b. Find the radius of curvature of the curve $y = x^3(x-a)$ at the point (a,o). **(04 Marks)**

c. State and prove Lagrange's Mean value theorem. **(06 Marks)**

d. Expand $\sqrt{1+\sin 2x}$ using Maclaurin's series up to the term containing x^4. **(06 Marks)**

2 a. Select the correct answer in each of the following :

i) $\underset{x\to 0}{\text{Lt}} \frac{x}{\sqrt{1-\cos x}}$ is equal to,

A) $\frac{1}{2}$ B) 1 C) $\sqrt{2}$ D) None of these.

ii) If $rt - S^2 > 0$, $r < 0$ then f(a,b) is

A) Maximum value of f(x,y) B) Minimum value of f(x,y)

C) Saddle point D) None of these.

iii) If $L(x, y, z, \lambda) = f(x, y, z) + \lambda\phi(x, y, z)$ is called,

A) Particular function B) Auxilliary function

C) General function D) None of these

iv) In a plane triangle ABC, the maximum value of COSA COSB COSC is,

A) $\frac{3}{8}$ B) $\frac{1}{8}$ C) $\frac{5}{8}$ D) $\frac{25}{8}$ **(04 Marks)**

b. Evaluate $\underset{x\to\frac{\pi}{2}}{\text{Lt}} (\text{Sec}x)^{\text{Cot}x}$. **(04 Marks)**

c. Expand $e^x\log(1+y)$ by Maclaurin's theorem upto the third degree term. **(06 Marks)**

d. Find the minimum value of $x^2 + y^2 + z^2$, subject to the condition $ax + by + cz = p$. **(06 Marks)**

3 a. Select the correct answer in each of the following :

i) $\int_0^2\int_0^x (x+y)dxdy$ is equal to

A) 4 B) 3 C) 5 D) None of these

ii) $\int_0^a\int_0^b\int_0^c x^2y^2z^2\,dx\,dy\,dz$ is equal to

A) $\frac{abc}{3}$ B) $\frac{a^2b^2c^2}{27}$ C) $\frac{a^3b^3c^3}{27}$ D) $\frac{a^2b^2c^2}{9}$

iii) The value of $\beta\left(\frac{1}{2},\frac{1}{2}\right)$ is equal to

A) 3.1416 B) 5.678 C) 2 D) None of these

iv) The value of $\sqrt{(n+1)}$ is ,

A) n B) n + 1 C) (n+1)! D) n! **(04 Marks)**

b. Evaluate $\int_R\int xy(x+y)dxdy$ over the region between $y = x$ and $y = x^2$. **(04 Marks)**

c. Evaluate $\int_0^a\int_0^x\int_0^{x+y} e^{x+y+z}dzdydx$. **(06 Marks)**

d. Express $\int_0^{\pi/2}\sqrt{\cot\theta}\,d\theta$ in terms of Beta function and hence evaluate **(06 Marks)**

4 a. Select the correct answer in each of the following : **(04 Marks)**

i) Using the following integral, work done by a force $\vec{F}$ can be calculated

A) Line integral B) Surface integral C) Volume integral D) None of these

ii) The value of the line integral $\int_C (y^2dx + x^2dy)$ where C is the boundary of the square $-1 \le x \le 1$, $-1 \le y \le 1$ is

A) 0 B) 2(x+y) C) 4 D) $\frac{4}{3}$

iii) Gauss divergence theorem is a relation between

A) a line integral and a surface integral
B) a surface integral and a volume integral.
C) a line integral and a volume integral.
D) two volume integrals.

iv) The spherical co-ordinate system is

A) orthogonal B) not orthogonal C) co-planar D) non – coplanar

b. If $\vec{F} = 3xyi - y^2j$, evaluate $\int_c \vec{F}\cdot d\vec{r}$ along the curve $y = 2x^2$ in the xy plane from (00) to (1,2). **(04 Marks)**

c. Evaluate, by Green's theorem, $\int_C (xy+y^2)dx + x^2dy$, where C is bounded by $y = x$ and $y = x^2$. **(06 Marks)**

d. Express the vector $\vec{F} = ZI - 2xJ + yK$ in cylindrical co-ordinates. **(06 Marks)**

PART – B

5 a. Select the correct answer in each of the following : **(04 Marks)**

i) The general solution of $(D^2 + W^2)\, y = 0$ is

A) $y = c_1 \cos wx + c_2 \sin wx$ B) $y = c_1 e^{wx} + c_2 e^{-wx}$

C) $y = c_1 \sin wx + c_2 \cos wx$ D) None of these.

ii) The P.I. of the differential equation $y'' + y = \cos x$ is

A) $\frac{1}{2}\sin x$ B) $\frac{1}{2}\cos x$ C) $\frac{1}{2}x\cos x$ D) $\frac{1}{2}x\sin x$

iii) The complimentary function of $(D^2 + D + 1)y = 0$ is

A) $\left(\cos\frac{\sqrt{5}}{2}x + i\sin\frac{\sqrt{5}}{2}n\right)$ B) $\left(\cos\frac{\sqrt{3}}{2}x + i\sin\frac{\sqrt{3}}{2}n\right)e^{-x/2}$

C) $\left(\cos\frac{\sqrt{3}}{2}x + i\sin\frac{\sqrt{3}}{2}n\right)$ D) None of these.

iv) By the method of undetermined co-efficients y_p of $y'' + 3y^{l} + 2y = 12x^2$ is ,

A) $a+bx+cx^2$ B) $a + bx$ C) $ax+bx^2+cx^3$ D) None of these.

b. Solve $\frac{d^3y}{dx^3} + 4\frac{dy}{dx} = \sin 2x$. **(04 Marks)**

c. Solve the equation $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} - 3y = e^x\cos x$. **(06 Marks)**

d. Using the method of undetermined co-efficients, solve the equation :

$\frac{d^2y}{dx^2} + \frac{dy}{dx} - 2y = x + \sin x$ **(06 Marks)**

6 a. Select the correct answer in each of the following : **(04 Marks)**

i) By the method of variation of parameters the value of W is called,

A) The Demorgan's function B) Euler's function

C) Wronskian of the function D) None of these.

ii) The general solution of $(x^2D^2\text{-}xD)\, y = 0$ is

A) $y = c_1 + c_2e^x$ B) $y = c_1 + c_2x^2$ C) $y = c_1 + c_2 e^{-x}$ D) $y = c_1x + c_2x^2$

iii) To transform $x\frac{dy^2}{dx^2} - x\frac{dy}{dx} + y = \log x$ in to a linear differential equation with constant coefficient, put x =

A) e^t B) log t, C) e^{-t} D) None of these

iv) Solutions of the differential equation $y'' + y = 0$, satisfying the conditions $y(0) = 1$ and $y\left(\frac{\pi}{2}\right) = 2$ is,

A) 2cosx+sinx B) cosx - sinx C) cosx + 2sinx D) None of these.

b. Solve $x^2\frac{d^2y}{dx^2} - 4x\frac{dy}{dx} + 6y = \cos[2\log x]$. **(04 Marks)**

c. Solve $y'' - 2y' + 2y = e^x \tan x$ by using the method of variation of parameters. **(06 Marks)**

d. Solve the initial value problem $\frac{d^2y}{dx^2} + y = \sin(x + a)$ satisfying the condition $y(0) = 0$; $y'(0) = 0$. **(06 Marks)**

7 a. Select the correct answer in each of the following : **(04 Marks)**

i) Laplace transform of $t^n e^{at}$ is

A) $\frac{n!}{(S+a)^n}$ B) $\frac{(n+1)!}{(S+a)^{n+1}}$ C) $\frac{n!}{(S-a)^{n+1}}$ D) $\frac{(n+1)!}{(S-a)^{n+1}}$

ii) Laplace transform of $\sin^2 3t$ is

A) $\frac{3}{S^2+36}$ B) $\frac{6}{S(S^2+36)}$ C) $\frac{18}{S(S^2+36)}$ D) $\frac{18}{S^2+36}$

iii) Laplace transform of $f^1(t) =$

A) $SL\{f(t)\} - f(0)$ B) $F(s)$ C) $SL\{f(t)\} - f'(0)$ D) None of these

iv) A unit step function is defined as,

A) $u(t-a) = \begin{cases} 0 & t < a \quad a \geq 0 \\ 1 & t \geq a \end{cases}$ B) $t - a = 0$

C) $u(t-a) = \begin{cases} 0 & t > a \\ 1 & t \geq a \end{cases}$ D) None of these

b. Find the Laplace transform of $e^{-t}\cos^2 3t$. **(04 Marks)**

c. Find the Laplace transform of $\frac{\cos at - \cos bt}{t} + t\sin at$. **(06 Marks)**

d. Express the function $f(t) = \begin{cases} t^2 & 0 < t < 2 \\ 4t & 2 < t < 4 \\ 8 & t > 4 \end{cases}$

In terms of unit step function and find its Laplace transform. **(06 Marks)**

8 a. Select the correct answer in each of the following : **(04 Marks)**

i) Inverse Laplace transform of $\frac{1}{S^2+4S+13}$ is,

A) $\frac{1}{2}e^{-3t}\sin 3t$ B) $\frac{1}{3}e^{-2t}\sin 3t$ C) $\frac{1}{4}e^{-2t}\sin 3t$ D) $\frac{1}{2}e^{3t}\sin 3t$

ii) Inverse Laplace transform $\frac{\pi}{S^2+\pi^2}$ is,

A) Sin t B) $\sin \pi t$ C) $\cos \pi t$ D) None of these.

iii) Inverse Laplace transform of $\frac{S^2+3S+7}{S^3}$ is,

A) $1+3t+\frac{7t^2}{2}$ B) $13t+\frac{t^2}{2}$ C) $1-3t+7t^2$ D) None of these.

iv) $L^{-1}\left\{\frac{1}{S^n}\right\}$ is possible only when n is

A) $n > 1$ B) $n \geq -1$ C) $n = 1, 2 \ldots$ D) $n < 1$.

b. Find the inverse Laplace transform of $\frac{S}{(2S-1)(3S-1)}$. **(04 Marks)**

c. Using the convolution theorem, obtain the inverse Laplace transform of $\frac{S}{(S^2+a^2)^2}$. **(06 Marks)**

d. Solve the differential equation $\frac{d^2y}{dt^2} - \frac{3dy}{dt} + 2y = e^{3t}$ with $y(0) = 0 = y'(0)$, using Laplace transform method. **(06 Marks)**

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06MAT21

Second Semester B.E. Degree Examination, May/June 2010
Engineering Mathematics - II

Time: 3 hrs. Max. Marks:100

Note: *1. Answer any FIVE full questions, choosing at least two from each part.*
2. Answer all objective type questions only in OMR sheet page 5 of the answer booklet.
3. Answer to objective type questions on sheets other than OMR will not be valued.

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

PART – A

1 a. Select the correct answer in each of the following :

i) Curvature of a straight line is
A) ∞ B) zero C) Both A and B D) None of these.

ii) Radius of the curvature of the curve $\gamma = a \sin\theta$ at the pole is
A) $\frac{\pi}{2}$ B) $-\frac{a_n}{2}$ C) $\frac{a_n}{2}$ D) zero.

iii) If f(x) is continuous in the closed interval [a, b] differential in (a, b) then $\exists$ at least one value c of x in (a, b) such that $f'(c) =$
A) $\frac{f(b)-f(a)}{b-a}$ B) $\frac{f(b)+f(a)}{b+a}$ C) $\frac{f(b)-f(a)}{b+a}$ D) None of these

iv) Maclaurin's series expansion of $\log(1+x)$ is
A) $x-\frac{x^2}{2}+\frac{x^3}{3}-\frac{x^4}{4}+..........$ B) $x-\frac{x^2}{2!}+\frac{x^3}{3!}-\frac{x^4}{4!}+..........$
C) $x+\frac{x^2}{2}+\frac{x^3}{3}+\frac{x^4}{4}+..........$ D) $x+\frac{x^2}{2!}+\frac{x^3}{3!}+\frac{x^4}{4!}+..........$ **(04 Marks)**

b. Show that for the ellipse in the pedal form $\frac{1}{p^2}=\frac{1}{a^2}+\frac{1}{b^2}-\frac{1}{a^2b^2}$, the radius of the curvature at the point (p, r) is a^2b^2/p^3. **(04 Marks)**

c. Verify the Roller theorem for the function $f(x) = (x-a)^m(x-b)^n$, $x \in (a, b)$. **(06 Marks)**

d. Expand $\tan(\frac{\pi}{4}+x)$ using the Maclaurin's expansion upto the 4^{th} degree term. **(06 Marks)**

2 a. Select the correct answer in each of the following :

i) The basic fundamental indeterminate forms are
A) $\frac{0}{0}$ B) $\frac{\infty}{\infty}$ C) 0 D) both A and B

ii) The value of $\underset{x \to \pi/2}{\text{lt}} \frac{\log \sin x}{\left(\frac{\pi}{2}-x\right)^2}$ is
A) zero B) ½ C) – ½ D) - 2

iii) The necessary and sufficient condition for maximum and minimum is
A) $f_x(xy) = 0$ B) $f_y(xy) = 0$ C) $f_x(xy) = 0 = f_y(xy)$ D) None of these.

iv) In a plane triangle ABC, the maximum value of $\cos a \cos b \cos c$ is,
A) 3/8 B) 1/8 C) 5/8 D) 25/8. **(04 Marks)**

b. Evaluate $\lim_{x \to a}\left[2-\left(\frac{x}{a}\right)\right]^{\tan\left(\frac{\pi x}{2a}\right)}$ **(04 Marks)**

c. Expand $\tan^{-1}(y/x)$ about the point (1, 1) up to 2^{nd} degree term. **(06 Marks)**

d. Find the minimum value of $x^2 + y^2 + z^2$ subject to the condition $ax + by + cz = p$. **(06 Marks)**

3 a. Select the correct answer in each of the following :

i) Value of $\int_0^1 \int_x^{\sqrt{x}} xy\, dx$ is

A) zero B) $-\frac{1}{24}$ C) $\frac{1}{24}$ D) 24

ii) R is the region of xy plane bounded by the curves $y = y_1(x)$, $y = y_2(x)$ and line x = a, and x = b. Then $\iint_R f(xy)dxdy$ is

A) $\int_{y=y_1(x)}^{y_2(x)} \int_{x=a}^{b} f(xy)dy\, dx$ B) $\int_{x=a}^{b} \int_{y=y_1(x)}^{y_2(x)} f(xy)dxdy$

C) $\int_{x=a}^{b} \int_{y=y_1(x)}^{y_2(x)} f(x,y)dydx$ D) All are correct.

iii) $\iint_R dxdy$ represents

A) Area of the region in polar form B) Area of the region in Cartesian form

C) Both A and B D) None of these.

iv) The value of $\Gamma(n+1)$ is

A) $n\Gamma(n)$ B) n! C) (n – 1)! D) Both A and B. **(04 Marks)**

b. If A is the area of the rectangular region bounded by the lines x = 0, x = 1 and y = 0 , y = 2 Evaluate $\int_A (x^2 + y^2)\, dA$. **(04 Marks)**

c. With usual notations, prove that $\sqrt{x}\ \Gamma(2m) = 2^{2m-1}\Gamma(m)\Gamma(2m+1)$. **(06 Marks)**

d. Evaluate $\int_0^1 \int_x^{\sqrt{x}} xy\, dy\, dx$, by changing the order of integration. **(06 Marks)**

4 a. Select the correct answer in each of the following :

i) If $\vec{F}$ is the force acted upon by the particle moves from one end of a curve to the other end. Then the total work done by $\vec{F}$ is

A) $\int_c \vec{F} \times d\vec{r}$ B) $\int_c \vec{F} \cdot d\vec{r}$ C) $\int_c d\vec{r}$ D) None of these.

ii) The line integral of $\vec{F} = x^2 i + xyj$ from O(0, 0) to P(1, 1) along the straight line is

A) 1/3 B) – 1/3 C) 2/3 D) 4/3

iii) If $\partial N/\partial x$, $\partial M/\partial y$ are continuous functions, C is a simple closed curve enclosing the region R in the xy - plane. The Green's theorem states that

A) $\oint_c Mdx + Ndy = \iint_R \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right)dxdy$ B) $\oint_c Mdx + Ndy = \iint_R \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right)dxdy$

C) $\oint_c Mdx + Ndy = \iint_R \left(\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}\right)dxdy$ D) $\oint_c Mdx - Ndy = \iint_R \left(\frac{\partial M}{\partial x} - \frac{\partial N}{\partial y}\right)dxdy$

iv) The cylindrical co-ordinate system is
A) Not orthogonal B) Orthogonal C) Coplanam D) Non-coplanar. **(04 Marks)**

b. Find the total work done by the force represented by $\vec{F} = 3xyi - yj - 2zxk$, in moving a particle round the circle $x^2 + y^2 = 4$. **(04 Marks)**

c. Verify the Green's theorem for $\oint_c (xy + y^2)dx + x^2dy$, where c is the alosed curve of the region bounded by $y = x$ and $y = x^2$. **(06 Marks)**

d. Express the vector $\vec{A} = zi - 2xj + yk$, in cylindrical coordinates. **(06 Marks)**

PART – B

5 a. Select the correct answer in each of the following :
i) Solution of the differential equation $(D^2 - a^2)y$ is
A) $a_1e^{-ax} + c_2e^{ax}$ B) $(ax + b)e^{ax}$ C) $(c_1 + c_2x + c_3x^2)e^{ax}$ D) $(c_1x + c_2x^2)e^{ax}$
ii) Particular integral of the differential equation $(D^2 + 5D + 6)y = e^x$ is
A) e^x B) $e^x/12$ C) $e^x/30$ D) $e^x/6$.
iii) Complementary function of $y'' - 2y' + y = x\,e^x \sin x$ is
A) $c_1e^x + c_2e^{-x}$ B) $(c_1x + c_2)e^x$ C) $(c_1 + c_2x)e^{-x}$ D) None of these.
iv) Particular integral of $(D^2 - 4)\,y = \sin 3x$ is
A) 1/4 B) – 1/13 C) 1/5 D) None of these. **(04 Marks)**

b. Solve $(D^3 + D^2 + 4D + 4)y = 0$ **(04 Marks)**

c. Solve $y'' + 16y = x \sin 3x$. **(06 Marks)**

d. Solve $(D^2 - D - 2)y = 1 - 2x - 9e^{-x}$ by the method of undetermined coefficients. **(06 Marks)**

6 a. Select the correct answer in each of the following :
i) The wronskin of cos x and sin x is
A) 0 B) 1 C) 2 D) 4
ii) To transform $(1+x)^2\frac{d^2y}{dx^2} + (1+x)\frac{dy}{dx} + y = \sin 2[\log(1+n)]$ into a L.D.E. with constant coefficients put $(1+x) =$
A) e^t B) log x C) e^x D) t.
iii) The solution of the differential equation $y'' + 6y = 0$ satisfies the condition $y(0) = 1$ and $y(\pi/2) = 2$ is
A) cosx + 2sinx B) 2cosx + sinx C) cosx – sinx D) None of these.
iv) $c_1\cos ax + c_2\sin ax - \frac{x}{2a}\cos ax$ is the general solution of
A) $(D^2 + a^2)y = \sin ax$ B) $(D^2 - a^2)y = \sin ax$
C) $(D^2 + a^2)y = \cos ax$ D) $(D + a)y = \sin x$ **(04 Marks)**

b. Solve $x^2\frac{d^2y}{dx^2} - x\frac{dy}{dx} + y = \log x$. **(04 Marks)**

c. Solve $y'' - 3y' + 2y = \frac{1}{1+e^{-x}}$, by variation of parameter method. **(06 Marks)**

d. Solve $\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 6x = 0$. Give that $x(0) = 0$, $\frac{dx}{dt}(0) = 15$. **(06 Marks)**

7 a. Select the correct answer in each of the following :

i) Laplace transform of f(t), $t \geq 0$ is defined by

A) $\int_0^\infty e^{-st} f(t)dt$ B) $\int_0^\infty e^{st} f(t)dt$ C) $\int_0^\infty e^{-t} f(t)dt$ D) $\int_1^\infty e^{-st} f(t)dt$

ii) Laplace transform of cos at is

A) $\frac{a}{s^2+a^2}$ B) $\frac{s}{s^2+a^2}$ C) $\frac{1}{s^2+a^2}$ D) $\frac{s}{s^2-a^2}$

iii) $L^{-1}\left\{\frac{\bar{f}(s)}{s}\right\}$ is

A) $\int_0^t f(t)dt$ B) $\int_0^t \frac{f(t)}{t} dt$ C) t^n f(t). D) None of these.

iv) Laplace transform of $f'(t)$ is

A) s f(s) – f(0) B) s f′(s) – f(0) C) f(s) – f(0) D) None of these. **(04 Marks)**

b. Find L $\{e^{at} + 2t^n - 3 \sin 3t + 4 \cosh 2t\}$ **(04 Marks)**

c. If f(t) is a periodic function of period 'w', then show that

$$L\{f(t)\} = \frac{1}{1-e^{-sw}} \int_0^w e^{-st} f(t)dt$$ **(06 Marks)**

d. Express the function $f(t) = \begin{cases} \sin t & o < t \leq \pi/2 \\ \cos t & t > \pi/2. \end{cases}$, in terms of unit step function and find its Laplace transform. **(06 Marks)**

8 a. Select the correct answer in each of the following :

i) Inverse Laplace transform of $\frac{s-a}{(s-a)^2+b^2}$ is

A) e^tcosbt B) e^{at}cosbt C) e^{-at}cosbt D) e^{at}sinbt

ii) Inverse Laplace transform of $\left[\frac{s^2-3s+4}{s^3}\right]$ is

A) $1 - 3t + 2t^2$ B) $10 - 3t + 2t^2$ C) $4 - 3t + 4t^2$ D) None of these.

iii) L {u(t – a)}, where u (t – a) is a unit step function is

A) $\frac{e^{-as}}{a}$ B) $\frac{e^{as}}{s}$ C) e^{as} D) $s\,e^{-as}$

iv) L {δ (t – a)}, where δ(t – a) is n unit impulse function

A) e^{as} B) e^{-as} C) e^s D) $\frac{e^{-as}}{s}$ **(04 Marks)**

b. Find the inverse Laplace transform of $\frac{3s+2}{s^2-s-2}$. **(04 Marks)**

c. Using the convolution theorem obtain $L^{-1}\left\{\frac{s^2}{(s^2+a^2)(s^2+b^2)}\right\}$ **(06 Marks)**

d. Solve the differential equation $y''(t) + 4y'(t) + 4y(t) = e^{-t}$ with y(0) = 0 y′(0), using the Laplace transform method. **(06 Marks)**

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